

Hawking emission of gravitons in higher dimensions: non-rotating black holes

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We compute the absorption cross section and the total power carried by gravitons in the evaporation process of a higher-dimensional non-rotating black hole. These results are applied to a model of extra dimensions with standard model fields propagating on a brane. The emission of gravitons in the bulk is highly enhanced as the spacetime dimensionality increases. The implications for the detection of black holes in particle colliders and ultrahigh-energy cosmic ray air showers are briefly discussed.

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I. INTRODUCTION

Models with extra dimensions have emerged as the most successful candidates for a consistent unified theory of fundamental forces. The most recent formulation of superstring theory suggests that the five consistent string theories and 11-dimensional supergravity constitute special points in the moduli space of a more fundamental nonperturbative theory, called *M*-theory [1]. In models of warped or large extra dimensions, standard model fields but the graviton are confined to a three-dimensional membrane [2, 3]. The observed hierarchy between the electroweak and the gravitational coupling constants is naturally explained by the largeness of the extra dimensions.

The presence of extra dimensions changes drastically our understanding of high-energy physics and gravitational physics. Gravity in higher dimensions is much different than in four dimensions. For example, black holes with a fixed mass may have arbitrarily large angular momentum [4]. The uniqueness theorem does not hold, allowing higher-dimensional black holes with non-spherical topology. A naïve analysis suggests that higher-dimensional black holes should evaporate [5] more quickly than in four dimensions, due to the larger phase space. Moreover, brane emission should dominate over bulk emission because standard model fields carry a larger number of d.o.f. than the graviton [6]. However, a black hole does not radiate exactly as a black body and the emission spectrum depends crucially on the structure and dimensionality of the spacetime. A large graviton emissivity could reverse the above conclusion; if the probability of emitting spin-2 quanta is much higher than the probability of emitting lower spin quanta, the black hole could evaporate mainly in the bulk [7]. A conclusive statement on this issue can only be reached if the relative emissivities of all fields (greybody factors) are known. This is particularly relevant for the phenomenology of black holes in models of low-energy scale gravity [8, 9]. Unequivocal detection of subatomic black holes in particle colliders [10] and ultrahigh-energy cosmic ray observatories [11, 12] is only possible if a consistent fraction of the initial black hole mass [13, 14] is channeled into brane fields.

If the center-of-mass energy of the event is sufficiently large compared to the Planck scale, quantum gravitational effects can be neglected and the black hole can be treated classically. It is commonly accepted that black holes with masses larger than few Planck masses satisfy this criterion. Under this assumption, if the fundamental gravitational scale is about a TeV, micro black holes produced at the Large Hadron Collider (center-of-mass energy = 14 TeV) and in ultrahigh-energy cosmic ray showers (center-of-mass energy $\gtrsim 50$ TeV) can be considered classical. Throughout the paper we will assume that this is the case. (See Ref. [12] and references therein for a discussion of the uncertainties

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deriving from this assumption.)

The relative emissivities per degree of freedom (d.o.f.) of a classical four-dimensional non-rotating black hole are 1, 0.37, 0.11 and 0.01 for spin-0, -1/2, -1 and -2, respectively [15]. In that case, the graviton power loss is negligible compared to the loss in other standard model channels. Since brane fields are constrained in four dimensions, the relative greybody factors for these fields approximately retain the above values in higher dimensions. The emission rates for the fields on the brane have been computed in Ref. [16]. The graviton emission is expected to be larger in higher dimensions due to the increase in the number of its helicity states. The authors have recently calculated the exact absorption cross section, power and emission rate for gravitons in generic D -dimensions [17]. (See also Ref. [18].) The purpose of this paper is to discuss these results in more detail. The graviton emissivity is found to be highly enhanced as the spacetime dimensionality increases. Although this increase is not sufficient to lead to a domination of bulk emission over brane emission, at least for the standard model, a consistent fraction of the higher-dimensional black hole mass is lost in the bulk.

The organization of this paper is as follows. In Section II we fix notations and briefly review the basics of gravitational perturbations in the higher-dimensional black hole geometry. In Section III we derive the field absorption cross sections from the absorption probabilities. Details of this derivation are included in the Appendix. The low-energy absorption probabilities and the cross sections for spin-0, -1 and -2 fields in generic dimensions are computed in Section IV. This derivation is valid for scalars, vectors and gravitons in the bulk and generalizes previous results [16]. In Section V we prove that the high-energy behavior of the cross section is universal and reduces to the capture cross section for a point particle. Numerical results for the total power and the emission rates for all known particle species are obtained in Section VI. Finally, Section VII contains our conclusions.

II. EQUATIONS AND CONVENTIONS

The formalism to handle gravitational perturbations of a higher-dimensional non-rotating black hole was developed by Kodama and Ishibashi [19] (hereafter, KI). In this section we briefly review their main results.

A. Number of degrees of freedom of gravitational waves

In four dimensions, gravitational waves have two possible helicities [20], corresponding to the number of spatial directions transverse to the propagation axis. In generic D dimensions, gravitational waves can be described by a symmetric traceless tensor of rank $D - 2$, corresponding to the $\square\square$ representation of $SO(D - 2)$. The number of helicities is

$$\mathcal{N} = \frac{(D - 2)(D - 1)}{2} - 1 = \frac{D(D - 3)}{2}. \quad (2.1)$$

This representation is decomposed into tensor (T), vector (V), and scalar (S) perturbations on the sphere S^{D-2} . These components correspond to symmetric traceless tensors, vectors and scalars, respectively. Tensors and vectors are divergenceless on S^{D-2} . The number of d.o.f. are

$$\mathcal{N}_T = \left(\frac{(D - 2)(D - 1)}{2} - 1 \right) - (D - 2), \quad \mathcal{N}_V = (D - 2) - 1, \quad \mathcal{N}_S = 1. \quad (2.2)$$

The perturbations are further expanded in tensor, vector and spherical harmonics on S^{D-2} . The total number of helicity states is obtained by considering the multiplicities of these components. A massless particle of spin J in four dimensions has helicities $+J$ and $-J$, corresponding to the projections of the spin along the direction of motion. These states provide a representation of the little group $SO(D - 2) = SO(2)$, i.e. the group of spatial rotations preserving the particle direction of motion. In four dimensions all massless particles have two helicities since all non-singlet representations of $SO(2)$ have dimension two. The description of spin in D dimensions proceeds similarly to the four-dimensional case. For instance, in five dimensions there are three directions orthogonal to the direction of motion. The little group is $SO(3)$ and the helicities of the 5-dimensional graviton are 2, 1, 0, -1, -2. For a general discussion, see Ref. [21].

B. Metric and master equations for gravitational perturbations

The metric of the higher-dimensional non-rotating spherically symmetric black hole is [4, 22]

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_{D-2}^2, \quad (2.3)$$

where

$$f = 1 - \frac{r_H}{r^{D-3}}. \quad (2.4)$$

The mass of the black hole is $M = (D-2)\Omega_{D-2}r_H/(16\pi\mathcal{G})$, where Ω_{D-2} is the volume of the unit $(D-2)$ -dimensional sphere with line element $d\Omega_{D-2}^2$ and \mathcal{G} is Newton's constant in D dimensions. Without loss of generality, we choose $r_H = 1$. This amounts to rescaling the radial coordinate and ω . Hereafter, ω stands for ωr_H .

Let us consider weak perturbations of the geometry (2.3) by external fields. If the field is a scalar, the contribution to the total energy-momentum tensor is negligible. The evolution for the field is simply given by the Klein-Gordon equation in the fixed background. The evolution equations for the electromagnetic field were derived by Crispino, Higuchi and Matsas [23] and more recently by KI in the context of charged black hole perturbations. According to the KI formalism, the perturbations are divided into vector and scalar perturbations. The gravitational evolution equations were derived by KI [19] following earlier work by Regge and Wheeler [24] and Zerilli [25] in four dimensions. The gravitational perturbations are divided in scalar, vector and tensor perturbations. Tensor perturbations exist only in $D > 4$.

The evolution equation for all known fields (scalar, electromagnetic and gravitational) can be reduced to the second order differential equation

$$\frac{d^2\Psi}{dr_*^2} + (\omega^2 - V)\Psi = 0, \quad (2.5)$$

where r is a function of the tortoise coordinate r_* , which is defined by $\partial r/\partial r_* = f(r)$. With the exception of gravitational scalar perturbations, the potential V in Eq. (2.5) can be written as

$$V = f(r) \left[\frac{l(l+D-3)}{r^2} + \frac{(D-2)(D-4)}{4r^2} + \frac{(1-p^2)(D-2)^2}{4r^{D-1}} \right]. \quad (2.6)$$

The constant p depends on the field under consideration:

$$p = \begin{cases} 0 & \text{for scalar and gravitational tensor perturbations,} \\ 2 & \text{for gravitational vector perturbations,} \\ 2/(D-2) & \text{for electromagnetic vector perturbations,} \\ 2(D-3)/(D-2) & \text{for electromagnetic scalar perturbations.} \end{cases} \quad (2.7)$$

For radiative modes, the angular quantum number l takes the integer values

$$l = \begin{cases} 0, 1, \dots & \text{for scalar perturbations,} \\ 1, 2, \dots & \text{for electromagnetic perturbations,} \\ 2, 3, \dots & \text{for gravitational perturbations.} \end{cases} \quad (2.8)$$

The potential for gravitational scalar perturbations is

$$V = f \frac{Q(r)}{16r^2 H(r)^2}, \quad (2.9)$$

where

$$Q(r) = (D-2)^4(D-1)^2x^3 + (D-2)(D-1) \quad (2.10)$$

$$\times \{4[2(D-2)^2 - 3(D-2) + 4]m + (D-2)(D-4)(D-6)(D-1)\}x^2 \\ - 12(D-2)\{(D-6)m + (D-2)(D-1)(D-4)\}mx + 16m^3 + 4D(D-2)m^2,$$

$$H(r) = m + \frac{1}{2}(D-2)(D-1)x, \quad m = l(l+D-3) - (D-2), \quad x \equiv \frac{1}{r^{D-3}}, \quad (2.11)$$

and $l \geq 2$ has been assumed. In four dimensions, the equations for scalar and vector perturbations reduce to the Zerilli and Regge-Wheeler equations, respectively. In the limit $l \rightarrow \infty$, Eq. (2.5) shows the universal behavior

$$\frac{d^2\Psi}{dr_*^2} + (\omega^2 - f \frac{l^2}{r^2})\Psi = 0, \quad l \rightarrow \infty. \quad (2.12)$$

The above equation guarantees that all perturbations behave identically in this limit. The effective potential for tensor perturbations is equal to the potential of a massless scalar field in the higher-dimensional Schwarzschild black hole background [26].

Assuming a harmonic wave $e^{i\omega t}$ and ingoing waves near the horizon, we impose the boundary condition

$$\Psi(r) \rightarrow e^{-i\omega r_*} \sim (r-1)^{-i\omega/(D-3)}, \quad r \rightarrow r_+. \quad (2.13)$$

At the asymptotic infinity, the wave behavior includes both ingoing and outgoing waves

$$\Psi(r) \rightarrow Te^{-i\omega r_*} + Re^{i\omega r_*}, \quad r_* \rightarrow \infty. \quad (2.14)$$

The physical picture is a wave with amplitude T scattering on the black hole. Part of this wave is reflected back with amplitude $|R|^2$ and part is absorbed with probability $|\mathcal{A}|^2 = (|T|^2 - |R|^2)/|T|^2$.

III. FROM ABSORPTION PROBABILITIES TO ABSORPTION CROSS SECTIONS

The rate of absorbed particles for a plane wave of spin s and flux Φ^s is

$$\frac{dN_s}{dt} = \sigma^s \Phi^s, \quad (3.1)$$

where σ^s is the absorption cross section. In Eq. (3.1) we sum over all final states and average over the initial states. The total cross section is obtained by summing the absorption coefficients for each single mode l , \mathcal{A}_l^s , weighted by the multiplicity factors. For a scalar field, the result is ¹

$$\sigma^{s=0}(D, \omega) = C^{s=0}(D, \omega) \sum_l N_{lS} |\mathcal{A}_{lS}^{s=0}|^2, \quad (3.2)$$

where the normalization factor is

$$C^{s=0}(D, \omega) = \left(\frac{2\pi}{\omega}\right)^{D-2} \frac{1}{\Omega_{D-2}} = \frac{(4\pi)^{(D-3)/2} \Gamma[(D-1)/2]}{\omega^{D-2}}, \quad (3.3)$$

and the multiplicities of the scalar spherical harmonics are

$$N_{lS} = \frac{(2l+D-3)(l+D-4)!}{l!(D-3)!}. \quad (3.4)$$

Equation (3.3) follows from the decomposition of a plane wave in spherical waves. In four dimensions, Eq. (3.4) gives the well-known result $N_{lS} = 2l+1$.

Spherical harmonics and partial wave expansion are different for vector and tensor fields. This leads to different expressions for the normalization and multiplicity factors. (This fact has been overlooked in the literature. See, for instance, Ref. [9].)

Consider a vector field V^μ in the transverse gauge. The decomposition of a transverse plane wave in spherical waves gives the factor (see the Appendix and Ref. [23])

$$C^{s=1}(D, \omega) = \frac{C^{s=0}(D, \omega)}{D-2}. \quad (3.5)$$

Following KI, we first decompose V^μ in the divergence of a scalar field plus a divergenceless vector. These components are then expanded in scalar and vector spherical harmonics. The multiplicities of the scalar harmonics are given by Eq. (3.4). The multiplicities of the vector harmonics are

$$N_{lV} = \frac{l(l+D-3)(2l+D-3)(l+D-5)!}{(l+1)!(D-4)!}. \quad (3.6)$$

The cross section is

$$\sigma^{s=1} = C^{s=1}(D, \omega) \sum_l \left[N_{lS} |\mathcal{A}_{lS}^{s=1}|^2 + N_{lV} |\mathcal{A}_{lV}^{s=1}|^2 \right]. \quad (3.7)$$

¹ See, for example, Ref. [27] for the four-dimensional case and the Appendix and Ref. [9, 28] for its D -dimensional generalization.

Finally, let us consider the graviton field. The decomposition of a plane wave in spherical waves yields the normalization factor (see the Appendix for details)

$$C^{s=2}(D, \omega) = \frac{2}{D(D-3)} C^{s=0}(D, \omega). \quad (3.8)$$

The gravitational perturbation is first decomposed in scalar, vector and tensor components on the tangent space of S^{D-2} . These components are then expanded in scalar, vector and tensor harmonics. The corresponding multiplicities are N_{lS} , N_{lV} and [29]

$$N_{lT} = \frac{1}{2} \frac{(D-1)(D-4)(l+D-2)(l-1)(2l+D-3)(l+D-5)!}{(l+1)!(D-3)!}, \quad (3.9)$$

respectively. The graviton absorption cross section is

$$\sigma^{s=2} = C^{s=2}(D, \omega) \sum_l \left[N_{lS} |\mathcal{A}_{lS}^{s=2}|^2 + N_{lV} |\mathcal{A}_{lV}^{s=2}|^2 + N_{lT} |\mathcal{A}_{lT}^{s=2}|^2 \right]. \quad (3.10)$$

Note that in $D = 4$, accidentally, the conversion factors do not depend on the spin. The four-dimensional cross sections read

$$\sigma^s(4, \omega) = \frac{\pi}{\omega^2} \sum_l (2l+1) |\mathcal{A}_l^s|^2, \quad s = 0, 1, 2. \quad (3.11)$$

(See also the discussion in Section V below.)

IV. LOW-ENERGY ABSORPTION PROBABILITIES

Since low frequencies ($\omega \ll 1$) give a substantial contribution to the total power emission, it is worthwhile to derive an analytical expression for this limiting case. The low-energy approximation uses a matching procedure to find a solution on the whole spacetime for any value of p . Let us define the near-horizon region $r - 1 \ll 1/\omega$ and the asymptotic region $r \gg 1$. To solve the wave equation in the near-horizon region, we set

$$\Psi = r^{\beta^0/2} \Xi, \quad (4.1)$$

where $\beta^0 = (D-2)(p+1)$. With this substitution, the wave equation (2.5) is cast in the form

$$\frac{f}{r^{\beta^0}} \partial_r \left(f r^{\beta^0} \partial_r \Xi \right) + \left[\omega^2 + \frac{f}{r^2} \left(-l(l+D-3) + \frac{p(D-3)(D-2)}{2} + \frac{p^2(D-2)^2}{4} \right) \right] \Xi = 0. \quad (4.2)$$

Changing variables to

$$v = 1 - \frac{1}{r^{D-3}}, \quad (4.3)$$

Eq. (4.2) reads

$$(1-v)^2 v^2 (3-D)^2 \partial_v^2 \Xi - (3-D)v(1-v)^2 \left(\frac{p(D-2)v}{(1-v)} - (3-D) \right) \partial_v \Xi + ((\omega r)^2 + va) \Xi = 0, \quad (4.4)$$

where

$$a = -l(l+D-3) + \frac{p(D-3)(D-2)}{2} + \frac{p^2(D-2)^2}{4}. \quad (4.5)$$

This equation can be put in a standard hypergeometric form by defining

$$p_h = \frac{p(D-2)}{D-3}, \quad \omega_h = \frac{\omega}{D-3}, \quad a_h = \frac{a}{(D-3)^2}, \quad (4.6)$$

and setting

$$\Xi = v^{c_1} (1-v)^{c_2} \mathcal{F}, \quad (4.7)$$

$$c_1 = -i\omega_h, \quad (4.8)$$

$$c_2 = \frac{1}{2} (1 + p_h - b) = \frac{1}{2} \frac{\beta^0 - 1 - (D-3)b}{D-3}, \quad (4.9)$$

$$b = \sqrt{(1+p_h)^2 - 4a_h - 4\omega_h^2} = \frac{1}{D-3} \sqrt{(D+2l-3)^2 - 4\omega^2}. \quad (4.10)$$

The result is

$$v(1-v)\partial_v^2 \mathcal{F} + (\gamma - v(1+\alpha+\beta)) \partial_v \mathcal{F} - \alpha\beta \mathcal{F} = 0, \quad (4.11)$$

where

$$\gamma = 1 - 2i\omega_h, \quad \alpha = \frac{1}{2} (1 - p_h - 2i\omega_h - b), \quad \beta = \frac{1}{2} (1 + p_h - 2i\omega_h - b). \quad (4.12)$$

The most general solution of Eq. (4.11) in the neighborhood of $v = 0$ is

$$\mathcal{F} = A F(\alpha, \beta, \gamma, v) + B v^{1-\gamma} F(\alpha - \gamma + 1, \beta - \gamma + 1, 2 - \gamma, v). \quad (4.13)$$

Since the second term describes an outgoing wave near the horizon, we set $B = 0$. The asymptotic behavior of the near-horizon solution is

$$\Xi \sim r^{-2i\omega - (\beta^0 - 1) - (D-3)b/2} \times \frac{\Gamma(\gamma)\Gamma(\alpha + \beta - \gamma)}{\Gamma(\alpha)\Gamma(\beta)} + r^{-(\beta^0 - 1) + (D-3)b/2} \frac{\Gamma(\gamma)\Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)}, \quad (4.14)$$

where we have used the property of the hypergeometric functions:

$$\begin{aligned} F(\alpha, \beta, \gamma, v) &= (1-v)^{\gamma-\alpha-\beta} \times \frac{\Gamma(\gamma)\Gamma(\alpha+\beta-\gamma)}{\Gamma(\alpha)\Gamma(\beta)} F(\gamma-\alpha, \gamma-\beta, \gamma-\alpha-\beta+1, 1-v) \\ &\quad + \frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)} F(\alpha, \beta, -\gamma+\alpha+\beta+1, 1-v). \end{aligned} \quad (4.15)$$

In the asymptotic region, Eq. (4.2) can be written as

$$\partial_r^2 \Xi + \frac{\beta^0}{r} \partial_r \Xi + \left[\omega^2 + \frac{f}{r^2} \left(-l(l+D-3) + \frac{p(D-3)(D-2)}{2} + \frac{p^2(D-2)^2}{4} \right) \right] \Xi = 0. \quad (4.16)$$

The solution of this equation is

$$\Xi = C_1 r^{(1-\beta^0)/2} J_{\hat{b}/2}(\omega r) + C_2 r^{(1-\beta^0)/2} Y_{\hat{b}/2}(\omega r), \quad (4.17)$$

where $\hat{b} = \sqrt{(1-\beta^0)^2 - 4a}$. Expanding Ξ for small ωr , we obtain

$$\Xi \sim C_1 \frac{(\omega/2)^{\hat{b}/2}}{\Gamma(1+\hat{b}/2)} r^{(1-\beta^0+\hat{b})/2} - C_2 \frac{(2/\omega)^{\hat{b}/2} \Gamma(\hat{b}/2)}{\pi} r^{(1-\beta^0-\hat{b})/2}. \quad (4.18)$$

Equation (4.18) is matched to Eq. (4.14). As $(D-3)b \sim \hat{b}$ for $\omega \ll 1$, we find

$$\frac{\Gamma(\gamma - \alpha - \beta)\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta - \gamma)\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)} = -\frac{C_1}{C_2} \frac{\pi(\frac{\omega}{2})^{\hat{b}}}{\Gamma(1+\frac{\hat{b}}{2})\Gamma(\frac{\hat{b}}{2})}, \quad (4.19)$$

$$\frac{C_1}{C_2} = -\left(\frac{2}{\omega}\right)^{D+2l-3} \frac{\Gamma(l + \frac{D-3}{2})^2 (l + \frac{D-3}{2}) \Gamma(\gamma - \alpha - \beta) \Gamma(\alpha) \Gamma(\beta)}{\pi \Gamma(\alpha + \beta - \gamma) \Gamma(\gamma - \alpha) \Gamma(\gamma - \beta)}, \quad (4.20)$$

$$\frac{C_1}{C_2} = -\left(\frac{2}{\omega}\right)^{D+2l-3} \frac{\Gamma(l + \frac{D-3}{2})^2 (l + \frac{D-3}{2}) \Gamma\left(1 + \frac{2l}{D-3}\right)^2 \Gamma(\beta)^2 \left(1 + \frac{2l}{D-3}\right) \sin \pi \beta \sin \frac{2l\pi}{D-3}}{\pi^2 \alpha^2 \sin \pi \alpha \Gamma(-\alpha)^2}. \quad (4.21)$$

It is straightforward to show that the reflection coefficient is

$$\mathcal{R} = \frac{R}{T} = \frac{C_1 - iC_2}{C_1 + iC_2}. \quad (4.22)$$

Therefore, the absorption probability in the $\omega \ll 1$ approximation is

$$|\mathcal{A}|^2 = 1 - |\mathcal{R}|^2 = 4\pi \left(\frac{\omega}{2}\right)^{D+2l-2} \frac{\Gamma\left(1 + \frac{2l+p(D-2)}{2(D-3)}\right)^2 \Gamma\left(1 + \frac{2l-p(D-2)}{2(D-3)}\right)^2}{\Gamma\left(1 + \frac{2l}{D-3}\right)^2 \Gamma\left(l + \frac{(D-1)}{2}\right)^2}. \quad (4.23)$$

The result for the scalar waves [16] are obtained by setting $p = 0$ and $l = 0, 1, \dots$. Setting $p = 0$ (2) and $l = 2, 3, \dots$ we obtain the low-energy absorption probability for the gravitational tensor (vector) perturbations. The gravitational scalar perturbation cannot be dealt analytically. Numerical results give

$$p_{\text{grav scalar}} \sim 2 + 0.674D^{-0.5445}. \quad (4.24)$$

Using Eqs. (3.4), (3.6), (3.8), (3.9) and (4.23) it is straightforward to compute the low-energy absorption cross section for the gravitational waves. For instance, recalling that the four-dimensional absorption probabilities of scalar and vector perturbations are equal [30] and the tensor contribution vanishes, the contribution of the $l = 2$ mode in four dimensions is

$$\sigma_l = \frac{4\pi}{45} \omega^4. \quad (4.25)$$

This result agrees with the well-known result of Ref. [15].

V. HIGH-ENERGY ABSORPTION CROSS SECTIONS

The wave equation can also be solved in the high-energy limit, where the absorption cross section is expected to approximate the cross section for particle capture. This has been verified in a number of papers for the scalar field. (See Ref. [9] and references therein.) We will now extend this result to spin-1 and -2 fields in any dimension D . In the high- l limit, the multiplicities satisfy the relations

$$N_{l,S} = \frac{1}{D-2} [N_{l,S} + N_{l,V}] = \frac{2}{D(D-3)} [N_{l,S} + N_{l,V} + N_{l,T}], \quad l \rightarrow \infty, \quad D > 2. \quad (5.1)$$

The above relations also hold for any l when $D = 4$. In that case, Chandrasekhar [30] showed that $\mathcal{A}_{lS} = \mathcal{A}_{lV}$ for any l and $N_{lT}(4, 2) = 0$. Therefore, in four dimensions the cross section does not depend on the particle spin.

Since high frequencies can easily penetrate the gravitational potential barrier, the absorption probabilities of all fields, $|\mathcal{A}(D, l, \omega)|^2$, tend to 1 as $\omega \rightarrow \infty$. A WKB analysis shows that the absorption probability is nonzero for $l \lesssim \omega$. Therefore, the cross section in the high-energy limit must include the contribution from all $l \lesssim \omega$. The largest contribution to the cross section when $\omega \rightarrow \infty$ is given by high- l modes. Since the high- l limit of the wave equation is independent of the kind of perturbation (see Section II B), the wave equation takes a universal form in this limit, i.e. $\mathcal{A}_{lS} = \mathcal{A}_{lV} = \mathcal{A}_{lT}$. From Eqs. (3.5)-(3.10) and (5.1) it follows that the absorption cross sections for spin-0, -1 and -2 fields is equal in the limit $\omega \rightarrow \infty$.

VI. TOTAL ENERGY EMISSION

The total energy flux for gravitational waves is

$$\frac{dE}{dt} = \frac{dE_S}{dt} + \frac{dE_V}{dt} + \frac{dE_T}{dt} = \sum_l \int \frac{d\omega}{2\pi} \frac{\omega}{e^{\omega/T_H} - 1} (N_{lS} |\mathcal{A}_{lS}^{s=2}|^2 + N_{lV} |\mathcal{A}_{lV}^{s=2}|^2 + N_{lT} |\mathcal{A}_{lT}^{s=2}|^2), \quad (6.1)$$

where the Hawking temperature is $T_H = (D-3)/(4\pi)$ and the counting of helicities is included in the multiplicity factors. The total absorption probabilities can be computed numerically. Equation (2.5) is integrated from a point near the horizon (typically $r-1 \sim 10^{-6}$), where the field behavior is given by Eq. (2.13). The numerical result is then

compared to Eq. (2.14) at large r . A better accuracy is achieved by considering the next-to-leading order correction terms [31]

$$\Psi(r) \rightarrow T \left(1 + \frac{\varepsilon}{r}\right) e^{-i\omega r_*} + R \left(1 - \frac{\varepsilon}{r}\right) e^{i\omega r_*}, \quad r_* \rightarrow \infty, \quad (6.2)$$

where

$$\varepsilon = -i \frac{l(l+D-3) + (D-2)(D-4)/4}{2\omega}. \quad (6.3)$$

The emission rates and the total integrated power for various fields are summarized in Tables I-III. For sake of comparison with previous works, the values for lower-spin fields are taken from Ref. [9]. (We checked these results with our numerical codes and found agreement within numerical uncertainties.) The results in the tables are normalized to the four-dimensional values. In four dimensions, the radiated power \mathcal{P} is

$$\mathcal{P}^{s=0} = 2.9 \times 10^{-4} r_+^{-2}, \quad \mathcal{P}^{s=1/2} = 1.6 \times 10^{-4} r_+^{-2}, \quad \mathcal{P}^{s=1} = 6.7 \times 10^{-5} r_+^{-2}, \quad \mathcal{P}^{s=2} = 1.5 \times 10^{-5} r_+^{-2}. \quad (6.4)$$

The spin-0, -1/2 and -1 values are per d.o.f., whereas the graviton value includes the contribution of the two helicities². The four-dimensional emission rates are

$$\mathcal{R}^{s=0} = 1.4 \times 10^{-3} r_+^{-1}, \quad \mathcal{R}^{s=1/2} = 4.8 \times 10^{-4} r_+^{-1}, \quad \mathcal{R}^{s=1} = 1.5 \times 10^{-4} r_+^{-1}, \quad \mathcal{R}^{s=2} = 2.2 \times 10^{-5} r_+^{-1}. \quad (6.5)$$

The above values for fermions, bosons and graviton agree with Page's results (See Table I in Ref. [15]). Some features

TABLE I: Total radiated power \mathcal{P} into different channels. The first three rows correspond to fields propagating on the brane. The last row gives the power radiated in bulk gravitons normalized to the four-dimensional case.

D	4	5	6	7	8	9	10	11
Scalars	1	8.94	36	99.8	222	429	749	1220
Fermions	1	14.2	59.5	162	352	664	1140	1830
Gauge Bosons	1	27.1	144	441	1020	2000	3530	5740
Gravitons	1	103	1036	5121	2×10^4	7.1×10^4	2.5×10^5	8×10^5

of the numerical results of Table I are worth discussing:

- The relative contributions of the higher partial waves increase with D . For instance, in four dimensions the contribution of the $l = 2$ mode is two orders of magnitude larger than the contribution of the $l = 3$ mode. The $l = 2$ and $l = 3$ contributions are roughly equal in $D = 8$. More energy is channeled in $l = 3$ mode than in $l = 2$ mode for $D \geq 9$. (The largest tensor contribution in ten dimensions comes from the $l = 4$ mode.) Contributions from high l are needed to obtain accurate results for large D . For instance, in ten dimensions the first 10 modes must be considered for a meaningful result. Therefore, precise values for very large D require the most CPU-time. The values in Table I have a 5% accuracy.
- The total power radiated in gravitons increases more rapidly with D than the power radiated in lower-spin fields. This is due to the increase in the multiplicity of the tensor perturbations, which is larger than the scalar multiplicity by a factor D^2 at high D . Therefore, the main contribution to the total power comes from tensor (and vector) modes. For instance, in ten dimensions the tensor modes contribute roughly half of the total power output.

Table II gives the fraction of radiated power per d.o.f. normalized to the scalar field. In four dimensions, the graviton channel is only about 5% of the scalar channel. Therefore, the power loss in gravitons is negligible compared to the power loss in lower-spin fields. This conclusion is reversed in higher dimensions. For instance, the graviton loss is about 35 times higher than the scalar loss in $D = 11$. Graviton emission is expected to dominate the black hole evaporation at very high D . The particle emission rates per d.o.f. are shown in Table III. The relative emission

² Since the number of graviton d.o.f. (helicities) depends on the spacetime dimensionality D , here and throughout the paper we give the *total* rate and the *total* power for the graviton field, rather than the rate and power *per d.o.f.*

TABLE II: Fraction of radiated power per d.o.f. normalized to the scalar field. The graviton d.o.f. (number of helicity states) are included in the results.

D	4	5	6	7	8	9	10	11
Scalars	1	1	1	1	1	1	1	1
Fermions	0.55	0.87	0.91	0.89	0.87	0.85	0.84	0.82
Gauge Bosons	0.23	0.69	0.91	1.0	1.04	1.06	1.06	1.07
Gravitons	0.053	0.61	1.5	2.7	4.8	8.8	17.7	34.7

TABLE III: Fraction of emission rates per d.o.f. normalized to the scalar field. The graviton result includes all the helicity states and counts as one d.o.f.

D	4	5	6	7	8	9	10	11
Scalars	1	1	1	1	1	1	1	1
Fermions	0.37	0.7	0.77	0.78	0.76	0.74	0.73	0.71
Gauge Bosons	0.11	0.45	0.69	0.83	0.91	0.96	0.99	1.01
Gravitons	0.02	0.2	0.6	0.91	1.9	2.5	5.1	7.6

rates of different fields can be obtained by summing on the brane d.o.f. For instance, the relative emission rates of standard model charged leptons (12 d.o.f.) and a 11-dimensional bulk graviton are roughly 1:1. This ratio becomes $\sim 40:1$ in five dimensions. To illustrate the relevance of these results for black hole in particle colliders, let us consider the minimal $U(1) \times SU(2) \times SU(3)$ standard model with three families and one Higgs field on a thin brane with fundamental Planck scale = 1 TeV. For black holes with mass \sim few TeV the Hawking temperature is generally above 100 GeV. The temperature of a six-dimensional black hole with mass equal to 5 (100) TeV is ~ 282 (133) GeV. The temperature increases with the spacetime dimension at fixed mass. Therefore, all d.o.f. can be considered massless. (Considering massive gauge bosons does not affect the conclusions significantly.) The spin-0, -1/2 and -1 d.o.f. on the brane are 4 (complex Higgs doublet), 90 (quarks + charged leptons + neutrinos) and 24 (massless gauge bosons), respectively. The relative emissivities for this model are shown in Table IV. Although the graviton emission is highly enhanced, the large number of brane d.o.f. implies that the brane channel dominates on the bulk channel. However, power loss in the bulk is significant and cannot be neglected at high D ; about 1/4 of the initial black hole mass is lost in the 11-dimensional bulk. This implies a larger-than-expected missing energy in particle colliders.

TABLE IV: Percentage of power going into each field species for the minimal $U(1) \times SU(2) \times SU(3)$ standard model with three families and one Higgs field above the spontaneous symmetry breaking scale. The four-dimensional results are taken from Ref. [15] and the higher dimensional results for fermions and gauge fields are taken from Ref. [9].

D	4	5	6	7	8	9	10	11
Scalars	6.8	4.0	3.7	3.6	3.6	3.5	3.3	2.9
Fermions	83.8	78.7	75.0	72.3	69.9	66.6	61.6	53.4
Gauge Bosons	9.3	16.7	20.0	21.7	22.3	22.2	20.7	18.6
Gravitons	0.1	0.6	1.3	2.4	4.2	7.7	14.4	25.1

VII. CONCLUSIONS

In this paper we have computed the absorption cross section and the total power carried by gravitons in the evaporation process of a higher-dimensional non-rotating black hole. We find that the power loss in the graviton channel is highly enhanced in higher-dimensional spacetimes. This has important consequences for the detection of microscopic black hole formation in particle colliders and ultrahigh-energy cosmic ray observatories, where a larger bulk emission implies larger missing energies and lower multiplicity in the visible channels. Despite the increase in graviton emissivity, for $4 < D \leq 11$ a non-rotating black hole in the Schwarzschild phase will emit mostly on the

brane due to the higher number of brane d.o.f.³ However, black hole energy loss in the bulk cannot be neglected in presence of extra dimensions. Graviton emission is expected to dominate the black hole evaporation at very high D .

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APPENDIX: DERIVATION OF THE CROSS SECTIONS

In this appendix we derive in detail the cross sections for spin-0, -1 and -2 fields in generic dimensions. The harmonic expansion is performed on the spherical manifold S^n with metric γ_{ab} and coordinates y^a ($a = 1, \dots, n = D - 2$). We also define the set of coordinates $x^i = (r, y^a)$. In the limit $r \rightarrow \infty$ the wave vector of a massless particle is

$$k^\mu = (\omega, \vec{k}), \quad (\text{A.1})$$

where $\vec{k} = (k^i) = \omega \hat{k}$. The direction of the plane wave is given by the $(n + 1)$ -dimensional unit vector \hat{k} .

Scalar perturbations

Let us consider an incoming scalar plane wave $\phi^{(\hat{k}, \omega)}$ with unit flux and wave vector $\vec{k} = \omega \hat{k}$ at infinity:

$$\phi^{(\hat{k}, \omega)} \rightarrow e^{-i\omega t - i\vec{k}\vec{x}}. \quad (\text{A.2})$$

This wave can be expanded in spherical waves $\Psi^{(\lambda, \omega)}$ parametrized by a discrete index λ :

$$\phi^{(\hat{k}, \omega)} = \sum_{\lambda} \alpha(\hat{k}, \omega; \lambda) \Psi^{(\lambda, \omega)}. \quad (\text{A.3})$$

In general, $\lambda = (l, m)$. However, we leave the index implicit to simplify the generalization to higher spins. The asymptotic behavior of the fields Ψ is

$$\Psi^{(\lambda, \omega)} \rightarrow \frac{e^{-i\omega t - i\omega r}}{r^{n/2}} Y^{(\lambda)}, \quad (\text{A.4})$$

where the spherical harmonics, $Y^{(\lambda)}$, satisfy the normalization and orthogonality conditions

$$\int d\Omega_n Y^{(\lambda)} Y^{(\lambda')} = \delta^{\lambda\lambda'}. \quad (\text{A.5})$$

From the above equations it follows

$$r^n \int d\Omega_n \Psi^{*(\lambda, \omega)} \Psi^{(\lambda, \omega)} = 1, \quad (\text{A.6})$$

i.e. the wave $\Psi^{(\lambda, \omega)}$ describes one ingoing particle per unit time. If $\mathcal{A}^{s=0}(\lambda, \omega)$ is the absorption coefficient for a spherical wave with angular profile $Y^{(\lambda)}$, the cross section associated to the plane wave (\hat{k}, ω) is

$$\sigma^{\hat{k}, \omega} = \sum_{\lambda} |\alpha(\hat{k}, \omega; \lambda)|^2 |\mathcal{A}^{s=0}(\omega, \lambda)|^2. \quad (\text{A.7})$$

³ This conclusion holds only if the particle content at TeV energies is mostly made of fields propagating on the brane. If non-standard model d.o.f. open up at the TeV scale, such as vector multiplets propagating in the bulk, black hole emission could occur mostly in the bulk.

The average on the wave direction of the initial state is obtained by integrating \hat{k} over the sphere and dividing by the unit volume Ω_n :

$$\sigma = \frac{1}{\Omega_n} \sum_{\lambda} \int d\Omega_n^{(\hat{k})} |\alpha(\hat{k}, \omega; \lambda)|^2 |\mathcal{A}^{s=0}(\omega, \lambda)|^2. \quad (\text{A.8})$$

From the previous equation, we obtain

$$C^{s=0}(n+2, \omega) = \frac{1}{\Omega_n} \int d\Omega_n^{(\hat{k})} |\alpha(\hat{k}, \omega; \lambda)|^2. \quad (\text{A.9})$$

The coefficients α are given by the coefficients of the expansion

$$\tilde{\phi}^{(\hat{k}, \omega)} = \sum_{\lambda} \alpha(\hat{k}, \omega; \lambda) \tilde{\Psi}^{(\lambda, \omega)}, \quad (\text{A.10})$$

where

$$\tilde{\Psi}^{(\lambda, \omega)} \equiv \frac{e^{-i\omega t - i\omega r}}{r^{n/2}} Y^{(\lambda)}, \quad \tilde{\phi}^{(\hat{k}, \omega)} \equiv e^{-i\omega t - i\vec{k}\vec{x}}, \quad (\text{A.11})$$

are defined in the flat $(n+1)$ -dimensional Euclidean space. From

$$\int d^{n+1}x \tilde{\Psi}^{*(\lambda', \omega')} \tilde{\Psi}^{(\lambda, \omega)} = 2\pi \delta(\omega - \omega') \delta_{\lambda\lambda'} \quad (\text{A.12})$$

it follows

$$(2\pi)^2 \delta(\omega - \omega') \delta(\omega' - \omega'') |\alpha(\hat{k}, \omega; \lambda)|^2 = \int d^{n+1}x d^{n+1}x' \tilde{\Psi}^{*(\lambda, \omega)}(x) \tilde{\phi}^{(\hat{k}, \omega')}(x) \tilde{\phi}^{*(\hat{k}, \omega'')}(x') \tilde{\Psi}^{(\lambda, \omega'')}(x'). \quad (\text{A.13})$$

Multiplying by ω'^n and integrating in $d\Omega_n^{(\hat{k})} d\omega'$, we obtain

$$\begin{aligned} & \omega^n (2\pi)^2 \delta(\omega - \omega'') \int d\Omega_n^{(\hat{k})} |\alpha(\hat{k}, \omega; \lambda)|^2 \\ &= \int d^{n+1}x d^{n+1}x' \tilde{\Psi}^{*(\lambda, \omega)}(x) \tilde{\Psi}^{(\lambda, \omega'')}(x') \left[\int d\omega' \omega'^n \int d\Omega_n^{(\hat{k})} e^{-i\vec{k}(\vec{x} - \vec{x}')} \right] \\ &= (2\pi)^{n+1} \int d^{n+1}x d^{n+1}x' \delta(\vec{x} - \vec{x}') \tilde{\Psi}^{*(\lambda, \omega)}(x) \tilde{\Psi}^{(\lambda, \omega'')}(x') = (2\pi)^{n+2} \delta(\omega - \omega''). \end{aligned} \quad (\text{A.14})$$

Finally, the normalization factor is

$$C^{s=0}(n+2, \omega) = \frac{1}{\Omega_n} \int d\Omega_n^{(\hat{k})} |\alpha(\hat{k}, \omega; \lambda)|^2 = \frac{1}{\Omega_n} \left(\frac{2\pi}{\omega} \right)^n. \quad (\text{A.15})$$

Vector perturbations

Let us consider an incoming transverse vector plane wave $\phi_{\mu}^{(p, \hat{k}, \omega)}$ with unit flux and wave vector at infinity:

$$\phi_{\mu}^{(p, \hat{k}, \omega)} \rightarrow e^{-i\omega t - i\vec{k}\vec{x}} \begin{pmatrix} 0 \\ \epsilon_i^p(\hat{k}) \end{pmatrix}, \quad (\text{A.16})$$

where $\epsilon_i^p(\hat{k})$ are the transverse polarization vectors ($p = 1, \dots, n$) and

$$\hat{k}^i \epsilon_i^p = 0, \quad \epsilon_i^p \epsilon_j^{p'} \delta^{ij} = \delta^{pp'}, \quad \sum_{p=1}^n \epsilon_i^p \epsilon_j^p = \delta_{ij} + \text{terms in } k_i, k_j. \quad (\text{A.17})$$

We assume x^i to be asymptotically Euclidean coordinates for simplicity. The wave can be expanded in spherical waves $\Psi^{(\lambda, \omega)}$ parametrized by a discrete index λ , which includes the polarizations:

$$\phi_\mu^{(p, \hat{k}, \omega)} = \sum_\lambda \alpha(p, \hat{k}, \omega; \lambda) \Psi_\mu^{(\lambda, \omega)}. \quad (\text{A.18})$$

The asymptotic fields Ψ_μ are

$$\Psi_\mu^{(\lambda, \omega)} \rightarrow \frac{e^{-i\omega t - i\omega r}}{r^{n/2}} \begin{pmatrix} 0 \\ 0 \\ r Y_a^{(\lambda)} \end{pmatrix}, \quad (\text{A.19})$$

where $Y_a^{(\lambda)}$ satisfy the normalization and orthogonality conditions

$$\int d\Omega_n Y_a^{(\lambda)} Y_b^{(\lambda')} \gamma^{ab} = \delta^{\lambda\lambda'}. \quad (\text{A.20})$$

From the above equation it follows

$$r^n \int d\Omega_n \Psi_\mu^{*(\lambda, \omega)} \Psi_\mu^{(\lambda, \omega)} g^{\mu\nu} = 1, \quad (\text{A.21})$$

i.e. a wave $\Psi_\mu^{(\lambda, \omega)}$ describes one ingoing particle per unit time. If $\mathcal{A}^{s=1}(\lambda, \omega)$ is the absorption coefficient for a spherical wave with angular profile $Y_a^{(\lambda)}$, the cross section associated to the plane wave (p, \hat{k}, ω) is

$$\sigma^{p, \hat{k}, \omega} = \sum_\lambda |\alpha(p, \hat{k}, \omega; \lambda)|^2 |\mathcal{A}^{s=1}(\omega, \lambda)|^2. \quad (\text{A.22})$$

The average on the wave direction and polarization of the initial state is obtained by summing on p , integrating \hat{k} over the sphere and dividing by the unit volume Ω_n and the number of polarizations n :

$$\sigma = \frac{1}{n\Omega_n} \sum_\lambda \int d\Omega_n^{(\hat{k})} \sum_{p=1}^n |\alpha(p, \hat{k}, \omega; \lambda)|^2 |\mathcal{A}^{(\omega, \lambda)}|^2. \quad (\text{A.23})$$

From the previous equation, we obtain

$$C^{s=1}(n+2, \omega) = \frac{1}{n\Omega_n} \int d\Omega_n^{(\hat{k})} \sum_{p=1}^n |\alpha(p, \hat{k}, \omega; \lambda)|^2. \quad (\text{A.24})$$

The coefficients α are the coefficients of the expansion

$$\tilde{\phi}_\mu^{(p, \hat{k}, \omega)} = \sum_\lambda \alpha(p, \hat{k}, \omega; \lambda) \tilde{\Psi}_\mu^{(\lambda, \omega)}, \quad (\text{A.25})$$

where

$$\tilde{\Psi}_i^{(\lambda, \omega)} \equiv \frac{e^{-i\omega t - i\omega r}}{r^{n/2}} \begin{pmatrix} 0 \\ 0 \\ r Y_a^{(\lambda)} \end{pmatrix}, \quad \tilde{\phi}_i^{(p, \hat{k}, \omega)} \equiv e^{-i\omega t - i\vec{k}\vec{x}} \epsilon_i^p(\hat{k}), \quad (\text{A.26})$$

are defined in the flat $(n+1)$ -dimensional Euclidean space. From

$$\int d^{n+1}x \tilde{\Psi}_i^{*(\lambda', \omega')} \tilde{\Psi}^i(\lambda, \omega) = 2\pi \delta(\omega - \omega') \delta_{\lambda\lambda'} \quad (\text{A.27})$$

it follows

$$(2\pi)^2 \delta(\omega - \omega') \delta(\omega' - \omega'') |\alpha(p, \hat{k}, \omega; \lambda)|^2 = \int d^{n+1}x d^{n+1}x' \tilde{\Psi}_i^{*(\lambda, \omega)}(x) \tilde{\phi}^i(p, \hat{k}, \omega')(x) \tilde{\phi}_j^{*(p, \hat{k}, \omega')}(x') \tilde{\Psi}^j(\lambda, \omega'')(x'). \quad (\text{A.28})$$

Multiplying by ω'^n , summing over p and integrating in $d\Omega_n^{(\hat{k})} d\omega'$, we obtain

$$\begin{aligned}
& \omega^n (2\pi)^2 \delta(\omega - \omega'') \int d\Omega_n^{(\hat{k})} \sum_{p=1}^n |\alpha(p, \hat{k}, \omega; \lambda)|^2 \\
&= \int d^{n+1}x d^{n+1}x' \tilde{\Psi}_i^{*(\lambda, \omega)}(x) \tilde{\Psi}^{j(\lambda, \omega'')}(x') \left[\int d\omega' \omega'^n \int d\Omega_n^{(\hat{k})} \left(\sum_{p=1}^n \epsilon^{i(p)}(\hat{k}) \epsilon_j^{*(p)}(\hat{k}) \right) e^{-i\vec{k}(\vec{x} - \vec{x}')} \right] \\
&= (2\pi)^{n+1} \int d^{n+1}x d^{n+1}x' \delta(\vec{x} - \vec{x}') \tilde{\Psi}_i^{*(\lambda, \omega)}(x) \tilde{\Psi}^{j(\lambda, \omega'')}(x') (\delta_j^i + \text{terms in } k^i, k_j) = (2\pi)^{n+2} \delta(\omega - \omega'').
\end{aligned} \tag{A.29}$$

Finally, the normalization factor is

$$C^{s=1}(n+2, \omega) = \frac{1}{n\Omega_n} \int d\Omega_n^{(\hat{k})} \sum_{p=1}^n |\alpha(p, \hat{k}, \omega; \lambda)|^2 = \frac{1}{n\Omega_n} \left(\frac{2\pi}{\omega} \right)^n. \tag{A.30}$$

Gravitational perturbations

In our normalizations, a spin-two field $\Phi_{\mu\nu}$ has number flux $\Phi_{\mu\nu}^* \Phi^{\mu\nu}$. (This choice is possible because the waves are monochromatic, i.e. $\sim e^{i\omega t}$.) If $\Phi_{\mu\nu}$ is multiplied by a suitable factor, it can be interpreted as a metric perturbation $h_{\mu\nu}$. The multiplication factor depends on ω and \hbar , but its explicit form is not relevant in the derivation of the cross section. An incoming transverse spin-two plane wave $\phi_{\mu\nu}^{(p, \hat{k}, \omega)}$ with unit flux and wave vector at infinity $\vec{k} = \omega \hat{k}$ is

$$\phi_{\mu\nu}^{(p, \hat{k}, \omega)} \rightarrow e^{-i\omega t - i\vec{k}\vec{x}} \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \epsilon_{ij}^p(\hat{k}) \end{array} \right), \tag{A.31}$$

where $\epsilon_{ij}^p(\hat{k})$ are the transverse traceless polarization vectors, $p = 1, \dots, \mathcal{N} = (n-1)(n+2)/2$ and

$$\hat{k}^i \epsilon_{ij}^p = 0, \quad \epsilon_{ij}^p \epsilon_{kl}^{p'} \delta^{ik} \delta^{jl} = \delta^{pp'}, \quad \sum_{p=1}^{\mathcal{N}} \epsilon_{ij}^p \epsilon^{pkl} = \delta_{(i}^k \delta_{j)}^l - \frac{1}{n} \delta_{ij} \delta^{kl} + \text{terms in } k_i, k_j, k^k, k^l. \tag{A.32}$$

This wave can be expanded in spherical waves $\Psi_{\mu\nu}^{(\lambda, \omega)}$

$$\phi_{\mu\nu}^{(p, \hat{k}, \omega)} = \sum_{\lambda} \alpha(p, \hat{k}, \omega; \lambda) \Psi_{\mu\nu}^{(\lambda, \omega)}. \tag{A.33}$$

The asymptotic fields $\Psi_{\mu\nu}$ are

$$\Psi_{\mu\nu}^{(\lambda, \omega)} \rightarrow \frac{e^{-i\omega t - i\omega r}}{r^{n/2}} \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & r^2 Y_{ab}^{(\lambda)} \end{array} \right), \tag{A.34}$$

where $Y_{ab}^{(\lambda)}$ satisfy the normalization and orthogonality conditions

$$\int d\Omega_n Y_{ab}^{(\lambda)} Y_{cd}^{(\lambda')} \gamma^{ac} \gamma^{bd} = \delta^{\lambda\lambda'}. \tag{A.35}$$

From the above equation it follows

$$r^n \int d\Omega_n \Psi_{\mu\nu}^{*(\lambda, \omega)} \Psi^{(\lambda, \omega) \mu\nu} = 1, \tag{A.36}$$

i.e. a wave $\Psi_{\mu\nu}^{(\lambda, \omega)}$ describes one ingoing particle per unit time. If $\mathcal{A}^{s=2}(\lambda, \omega)$ is the absorption coefficient for a spherical wave with angular profile $Y_{ab}^{(\lambda)}$, the cross section associated to the plane wave (p, \hat{k}, ω) is

$$\sigma^{p, \hat{k}, \omega} = \sum_{\lambda} |\alpha(p, \hat{k}, \omega; \lambda)|^2 |\mathcal{A}^{s=2}(\omega, \lambda)|^2. \tag{A.37}$$

The average on the wave direction and polarization of the initial state is obtained by summing on p , integrating \hat{k} on the sphere and dividing by the unit volume Ω_n and the number of polarizations \mathcal{N} :

$$\sigma = \frac{1}{\mathcal{N}\Omega_n} \sum_{\lambda} \int d\Omega_n^{(\hat{k})} \sum_{p=1}^{\mathcal{N}} |\alpha(p, \hat{k}, \omega; \lambda)|^2 |\mathcal{A}^{(\omega, \lambda)}|^2. \quad (\text{A.38})$$

From the previous equation, we obtain

$$C^{s=2}(n+2, \omega) = \frac{1}{\mathcal{N}\Omega_n} \int d\Omega_n^{(\hat{k})} \sum_{p=1}^{\mathcal{N}} |\alpha(p, \hat{k}, \omega; \lambda)|^2. \quad (\text{A.39})$$

The coefficients α are the coefficients of the expansion

$$\tilde{\phi}_{\mu\nu}^{(p, \hat{k}, \omega)} = \sum_{\lambda} \alpha(p, \hat{k}, \omega; \lambda) \tilde{\Psi}_{\mu\nu}^{(\lambda, \omega)}, \quad (\text{A.40})$$

where

$$\tilde{\Psi}_{ij}^{(\lambda, \omega)} \equiv \frac{e^{-i\omega t - i\omega r}}{r^{n/2}} \left(\frac{0}{0} \middle| \frac{0}{r^2 Y_{ab}^{(\lambda)}} \right), \quad \tilde{\phi}_{ij}^{(p, \hat{k}, \omega)} \equiv e^{-i\omega t - i\vec{k}\vec{x}} \epsilon_{ij}^p(\hat{k}) \quad (\text{A.41})$$

are defined in the flat $(n+1)$ -dimensional Euclidean space. From

$$\int d^{n+1}x \tilde{\Psi}_{ij}^{*(\lambda', \omega')} \tilde{\Psi}^{ij(\lambda, \omega)} = 2\pi \delta(\omega - \omega') \delta_{\lambda\lambda'}, \quad (\text{A.42})$$

it follows

$$(2\pi)^2 \delta(\omega - \omega') \delta(\omega' - \omega'') |\alpha(p, \hat{k}, \omega; \lambda)|^2 = \int d^{n+1}x d^{n+1}x' \tilde{\Psi}_{ij}^{*(\lambda, \omega)}(x) \tilde{\phi}^{ij(p, \hat{k}, \omega')}(x) \tilde{\phi}_{kl}^{*(p, \hat{k}, \omega'')}(x') \tilde{\Psi}^{kl(\lambda, \omega'')}(x'). \quad (\text{A.43})$$

Multiplying by ω^n , summing over p and integrating in $d\Omega_n^{(\hat{k})} d\omega'$, we obtain

$$\begin{aligned} & \omega^n (2\pi)^2 \delta(\omega - \omega'') \int d\Omega_n^{(\hat{k})} \sum_{p=1}^{\mathcal{N}} |\alpha(p, \hat{k}, \omega; \lambda)|^2 \\ &= \int d^{n+1}x d^{n+1}x' \tilde{\Psi}_{ij}^{*(\lambda, \omega)}(x) \tilde{\Psi}^{kl(\lambda, \omega'')}(x') \left[\int d\omega' \omega'^n \int d\Omega_n^{(\hat{k})} \left(\sum_{p=1}^{\mathcal{N}} \epsilon^{ij(p)}(\hat{k}) \epsilon_{kl}^{*(p)}(\hat{k}) \right) e^{-i\vec{k}(\vec{x} - \vec{x}')} \right] \\ &= (2\pi)^{n+1} \int d^{n+1}x d^{n+1}x' \delta(\vec{x} - \vec{x}') \tilde{\Psi}_{ij}^{*(\lambda, \omega)}(x) \tilde{\Psi}^{kl(\lambda, \omega'')}(x') \left(\delta_k^{(i} \delta_l^{j)} - \frac{1}{n} \delta^{ij} \delta_{kl} + \text{terms in } k^i, k^j, k_k, k_l \right) \\ &= (2\pi)^{n+2} \delta(\omega - \omega''). \end{aligned} \quad (\text{A.44})$$

Finally, the normalization factor is

$$C^{s=2}(n+2, \omega) = \frac{1}{\mathcal{N}\Omega_n} \int d\Omega_n^{(\hat{k})} \sum_{p=1}^{\mathcal{N}} |\alpha(p, \hat{k}, \omega; \lambda)|^2 = \frac{1}{\mathcal{N}\Omega_n} \left(\frac{2\pi}{\omega} \right)^n. \quad (\text{A.45})$$

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